Presentation

Estimating Mass Moments of Inertia – A Quick Check Method

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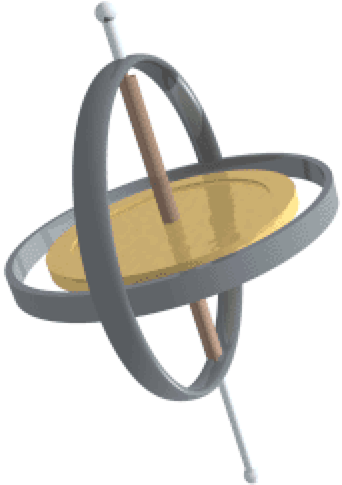
Overview

- What are Mass Moments of Inertia?
- Why are they important?
- What is Radius of Gyration?
- How can I quickly check MOI?
- What are the limitations of this method?
- Summary
- Questions?



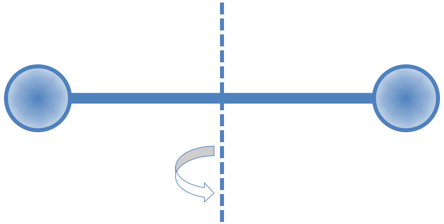
What are Mass Moments of Inertia (MOI)?

- **MOI is resistance to rotational acceleration or deceleration**

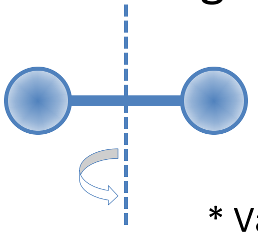
$$\boldsymbol{\tau = I\alpha}$$

Where $\boldsymbol{\tau}$ is torque, $\boldsymbol{I}$ is MOI, and $\boldsymbol{\alpha}$ is angular acceleration

- Units of mass x distance^2 (e.g. slug-ft^2, kg-m^2, lb-in^2*)
- MOI is dependent on the distribution of mass relative to the axis of rotation
  - The further distributed the mass is from the axis, the larger the MOI
  - Minimum when the axis of rotation passes through the CG



$$\boldsymbol{I_1 > I_2}$$



* Valid when magnitude of $g = g_c$

What are Mass Moments of Inertia (MOI)?

- **MOI is defined as**

$$I = \int r^2 dm$$

Where dm is an infinitesimal mass, and r is the distance of its CG from the axis of rotation

- The full inertia tensor, including products of inertia (POI), for a right hand, orthogonal coordinate system is:

$$I_{ij} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} = \begin{bmatrix} I_{xx} & -P_{xy} & -P_{xz} \\ -P_{xy} & I_{yy} & -P_{yz} \\ -P_{xz} & -P_{yz} & I_{zz} \end{bmatrix}$$

POI (Wobble)

$$P_{xy} = \int xydm, \ldots$$

MOI

Loads and Dynamics View

Mass Properties View

What are Mass Moments of Inertia (MOI)?

- **Summation Equations and Local Inertia Effects**

Total MOI (about axis parallel to X axis at total CG) | Local MOIs (about parallel axis at item CG) | Transfer Terms (to global reference) | Transfer Term (to total CG)

$$I_{oxx_T} = \sum_{i=1}^{n} I_{oxx_i} + \sum_{i=1}^{n} m_i({y_i}^2 + {z_i}^2) - m_T({y_T}^2 + {z_T}^2)$$

Where:

$$m_T = \sum_{i=1}^{n} m_i$$ — Total Mass

$$y_T = \frac{\sum_{i=1}^{n} m_i y_i}{m_T}$$ — Total CG in *y* direction

$$z_T = \frac{\sum_{i=1}^{n} m_i z_i}{m_T}$$ — Total CG in *z* direction

Similar for I_{oyy_T}, I_{ozz_T}

Increasing the discretization of masses in both the *y* and *z* directions decreases the effect of the local MOIs.

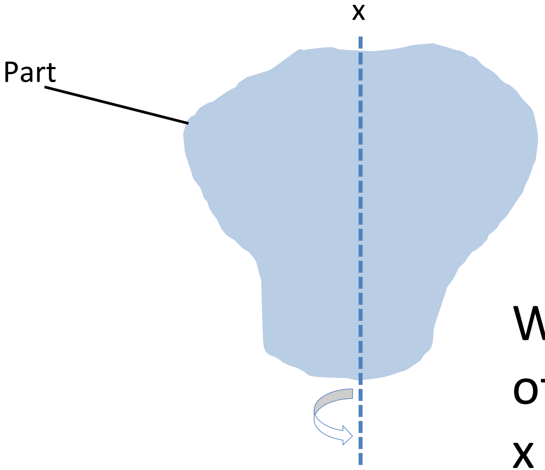
Why are MOI important?

- **MOI are essential for determining:**
  - Dynamic loads
  - Structural and system sizing
  - Performance
  - Maneuverability
  - Flutter prediction/avoidance

https://youtu.be/OhwLojNerMU

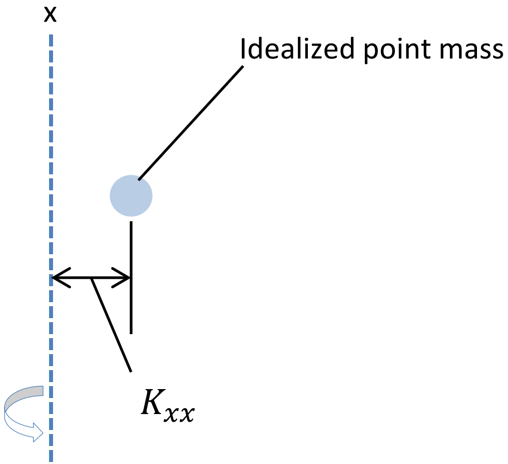
What is Radius of Gyration?

- **Radius of gyration (ROG) is the distance from the axis of rotation at which a point mass of equal value to the mass of the part could be placed to give equivalent rotational inertia.**
  - Easier to visualize than MOI
  - Dimension of length (e.g. inches, feet, meters, …)



$$I_{xx} = mK_{xx}{}^{2}$$

Where K_{xx} is the radius of gyration about the x axis.



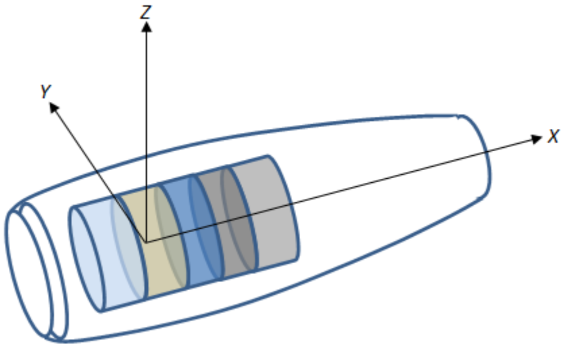
How can I quickly check MOI?

- **Quick Check Method**

The following steps provide some pointers for checking that the inertia values provided by a supplier are reasonable.

1. **Ensure that you understand the coordinate system and the units.**

Example: Aircraft Engine Assembly



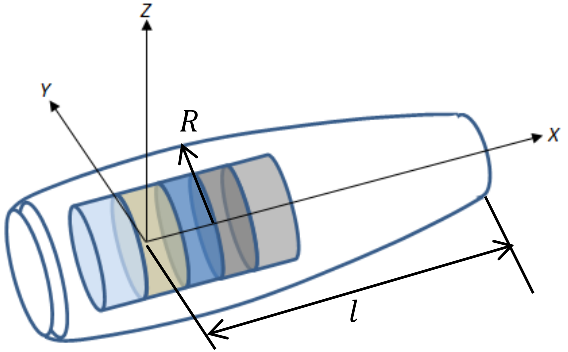
How can I quickly check MOI?

2. **Verify the overall radius of gyration (K_{ii}) about each axis as provided by the supplier is inside the spin radius boundary of the object.**

$$K_{ii} = \sqrt{I_{ii}/m}$$

Where: I_{ii} is the inertia about the x, y or z axis, and m is the mass (or weight*)

$$K_{xx} < R, K_{yy}\ and\ K_{zz} < l$$



* Valid when magnitude of $g = g_c$

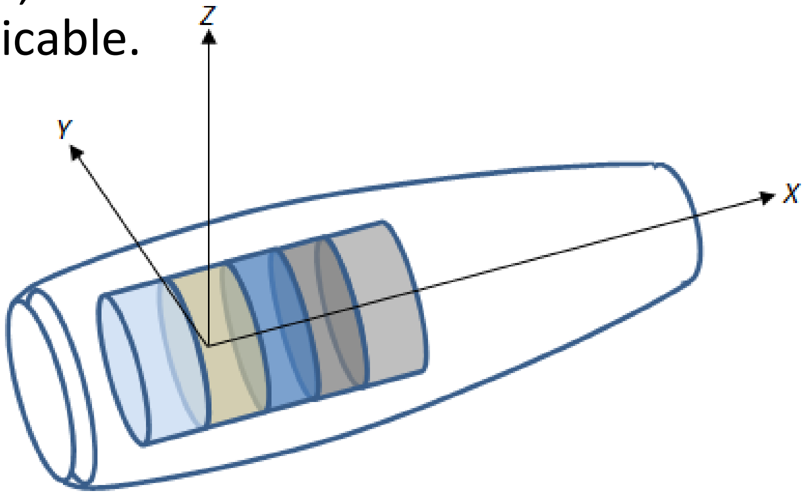
The radii of gyration cannot be larger than the spin radius boundaries!

How can I quickly check MOI?

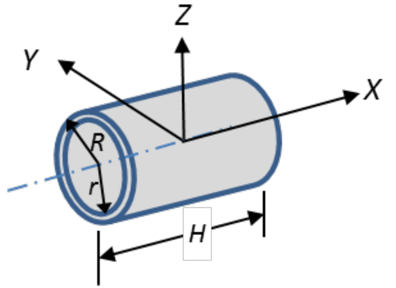
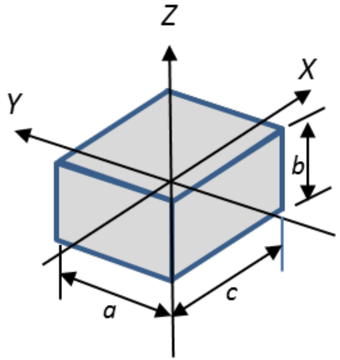
3. **Check that the magnitudes of the radii of gyration are reasonable relative to the overall dimensions and each other.**

(For this case, $K_{xx} < K_{yy}, K_{zz}$ and $K_{yy} \cong K_{zz}$)

- Use simple cylinder, rectangular prism, cone, sphere and shell shapes as applicable. (See SAWE Handbook)

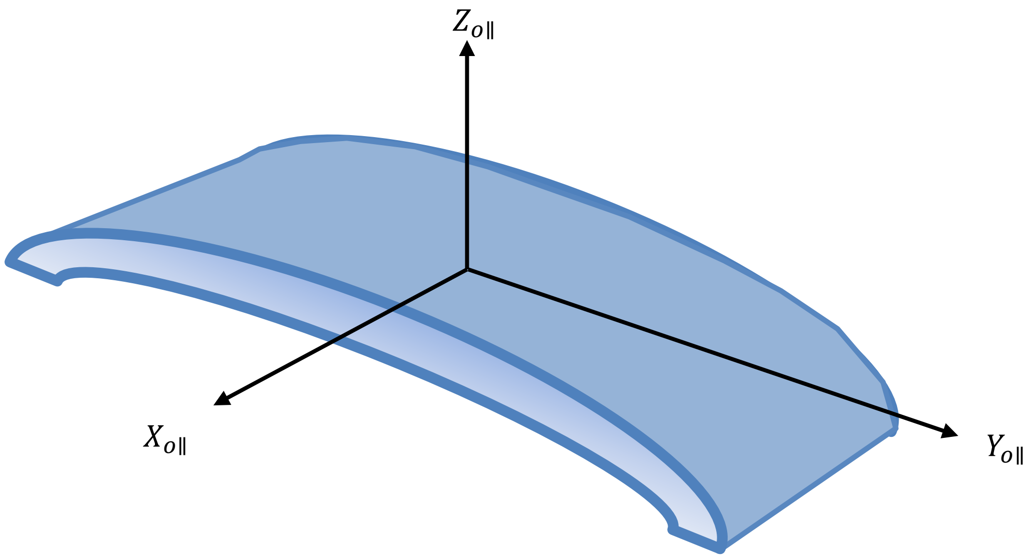


How can I quickly check MOI?

Cylinder	$K_{xx} = 0.707\sqrt{R^2 + r^2}$ $K_{yy} = K_{zz} = 0.289\sqrt{3(R^2 + r^2) + H^2}$ origin at the centroid of the shape	
Rectangular Prism	$K_{xx} = 0.289\sqrt{a^2 + b^2}$ $K_{yy} = 0.289\sqrt{b^2 + c^2}$ $K_{zz} = 0.289\sqrt{a^2 + c^2}$ or $K_{ii} = 0.289(diagonal)$ origin at the centroid of the shape	
Cone, Sphere and other shapes may be found in the SAWE Handbook.		

How can I quickly check MOI?

- **Example Part**



How can I quickly check MOI?

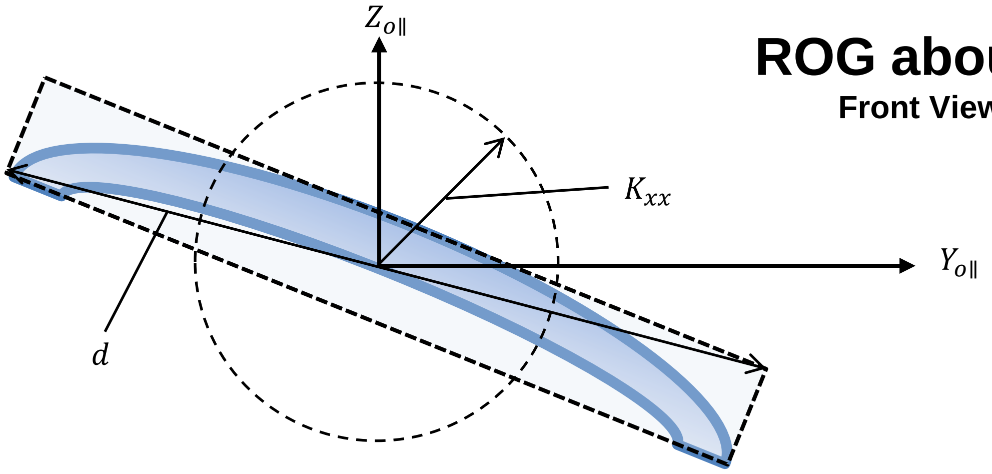
- **Orient the part so your line of sight is parallel to the desired axis.**
- **Visualize a box around the part.**
- **Determine the length of the diagonal of the box.**
- **Multiply diagonal by 0.289 to get the ROG.**
- **Calculate MOI**

$$I_{xx} = mK_{xx}^{2}$$

- **Repeat for the other two axes.**

ROG about X

Front View



X axis coming out of page

$$K_{xx} \cong 0.289d$$

This radius of gyration approximation will get you in the ballpark

How can I quickly check MOI?

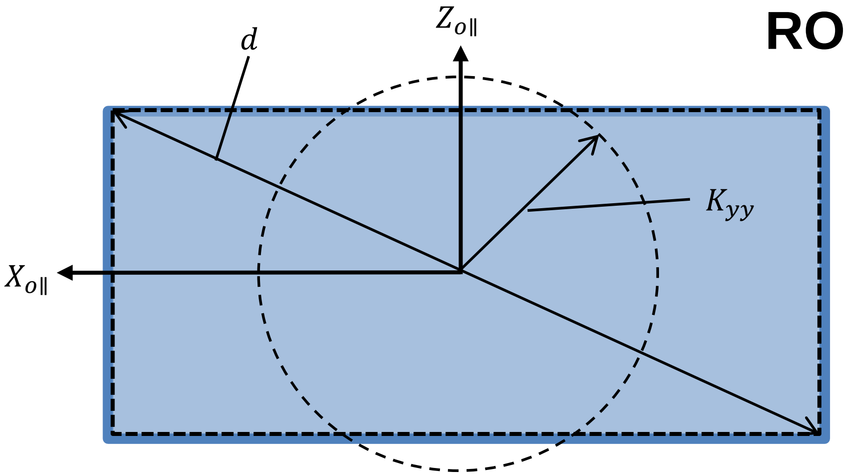
- **Orient the part so your line of sight is parallel to the desired axis.**
- **Visualize a box around the part.**
- **Determine the length of the diagonal of the box.**
- **Multiply diagonal by 0.289 to get the ROG.**
- **Calculate MOI**

$$I_{yy} = mK_{yy}^{2}$$

- **Repeat for the Z axis.**

ROG about Y

Side View



Y axis coming out of page

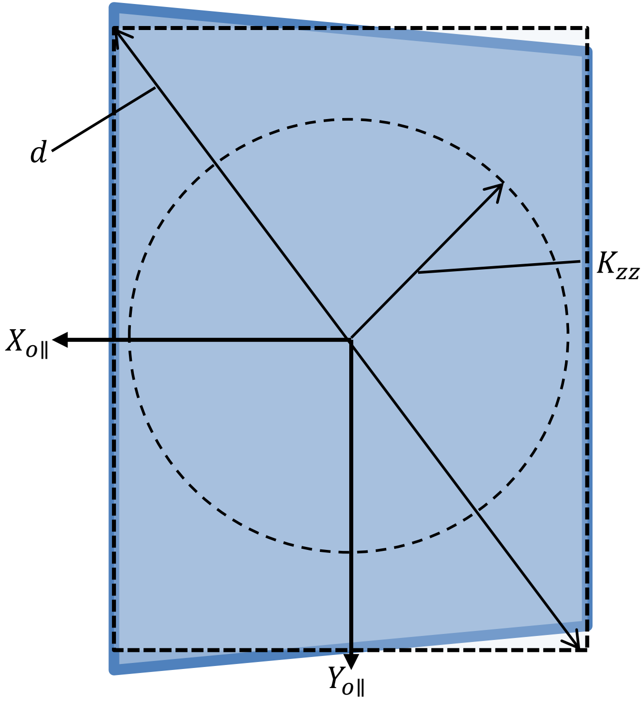
$$K_{yy} \cong 0.289d$$

This radius of gyration approximation will get you in the ballpark

How can I quickly check MOI?

- **Orient the part so your line of sight is parallel to the desired axis.**
- **Visualize a box around the part.**
- **Determine the length of the diagonal of the box.**
- **Multiply diagonal by 0.289**
- **Calculate MOI**

$$I_{zz} = mK_{zz}^2$$



ROG about Z
Top View

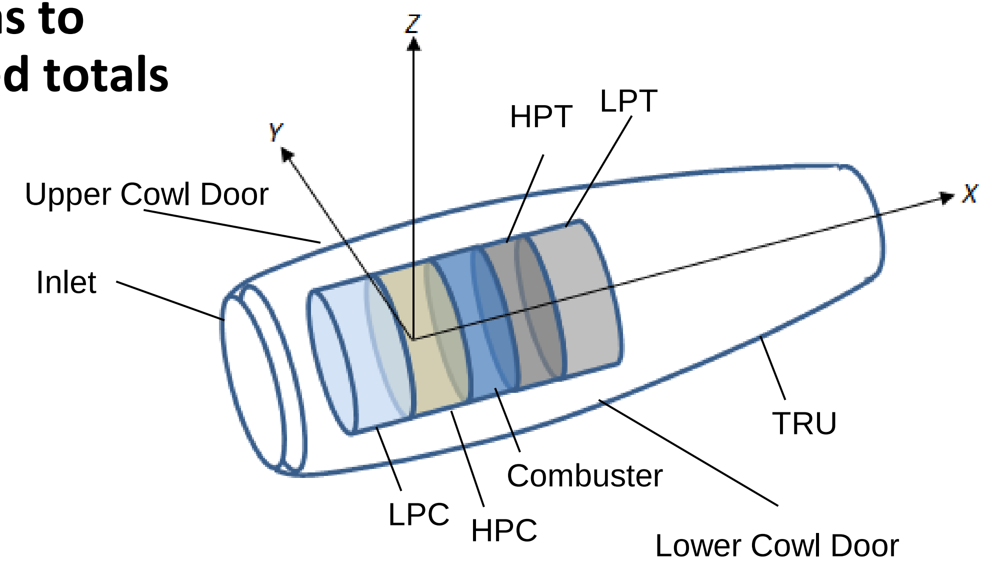
Z axis coming out of page

$K_{zz} \cong 0.289d$

This radius of gyration approximation will get you in the ballpark

How can I quickly check MOI?

4. Repeat Steps 2 thru 3 for each component to be checked, and question any ROGs that are different from your estimates by more than 10%. (=> MOI error 21%)
5. Check that the CGs seem reasonable.
6. Use the summation formulas to verify the supplier's reported totals for weight, CG and inertia.



What are the limitations of this method?

- **Assumptions that affect accuracy**
  - Parts are homogeneous solids.
  - CG is at the centroid of the shape.
- **Complex shapes and contours add uncertainty**
  - Consider breaking up the part into simpler shaped components.
- **Hollow structures have increased radii of gyration**
  - Consider using shell formulae, or subtracting inertia of removed volume.

Summary

- **MOI is the resistance to rotation**
- **Radius of Gyration is a very useful tool**
- **This estimation method is a quick way to get you in the ballpark**
  - $0.289(diagonal)$…
- **Use CAD analysis when available**
  - This method can be a quick check on the CAD analysis as well
  - Remember CAD results for off-diagonal terms (I_{xy}, I_{yz}, I_{xz}) are negative of POI (P_{xy}, P_{yz}, P_{xz})
- **Local MOI for <u>relatively</u> small parts can typically be ignored in rollup**
  - Effect of transfer terms will drive total MOI of assembly

Questions?

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